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The Minimum Total Heating Lander By The Maximum Principle Pontryagin

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ABSTRACT
The article will research a lander flying into the atmosphere with the flow velocity constraint, i.e. the total load by means of minimizing the total thermal energy at the end of the landing process. The lander's distance at the last moment depends on the variables selected from the total thermal energy minimum. To deal with the problem we apply Pontryagin maximum principle and scheme Dubovitsky- Milutin. Solving boundary using the parameter and the solution obtained in the choice of variables. The results of simulations performed on Matlab.
Keywords-maximum principle; control; the overload; total heat; minimum.

I. Introduction

Research on the problem of choosing an angle of launch of the flying object which is reducing velocity in atmospheric conditions under which it is taken into account the minimizing of total heat flow with the load limits of aircraft equipment. The total heat output of the device is the integral form of the following:

$$Q = \int_0^T CV^3 \rho^{1/2} dt \qquad (1)$$

Required to deteminea control $C_y(t)$, which minimizes Q(T) (1) under the following restrictions:

$$n_\Sigma = \sqrt{C_x^2 + C_y^2}\, q \frac{S}{G} \le N, \qquad q = \frac{\rho V^2}{2}, \qquad (2)$$

$$C_y^{\min} \le C_y \le C_y^{\max}, \qquad C_x = C_{xo} + kC_y^2 \qquad (3)$$

$$\rho = \rho_0 e^{-\beta H}, \; g = g_0 \frac{R^2}{(R+H)^2}, \; G = mg, \qquad (4)$$

$$\dot{V} = -C_x q \frac{S}{m} - g\sin\theta, \quad \dot{H} = V\sin\theta, \qquad (5)$$

$$\dot{\theta} = C_y q \frac{S}{mV} + \left(\frac{V}{R+H} - \frac{g}{V}\right)\cos\theta, \qquad (6)$$

$$\dot{L} = \frac{RV\cos\theta}{R+H}. \qquad (7)$$

Where n_Σ - full overload, q - speed pressure, ρ - atmospheric density, V - velocity of the vehicle, θ - path angle, H- height, L - the remote, G- the weight of the machine, m – mass, g_0 - acceleration due to gravity on the surface of the planet, R - the radius of the planet, C_x - the drag coefficient, C_y - lift coefficient, S - characteristic area apparatus,
$C_{xo},\ k,\ \rho_0,\ \beta,\ C,\ C_y^{\min},\ C_y^{\max},\ N$ - constants.

For the system (1) - (7) the initial conditions:

$$V(0) = V_0, \quad \theta(0) = \theta_0$$
$$H(0) = H_0, L(0) = L_0. \qquad (8)$$

and conditions and limitations:

$$V(T) = V_1, \quad \theta(T) = \theta_1, \quad H(T) = H_1,$$
$$L(T) = a, \; T \text{ - not fixed.} \qquad (9)$$

where a – parameter.

II. Application of Maximum principle in the regular case.

Let lander comes from the initial state (8) in a washed-position (9) in an optimal way in the sense of minimizing the total amount of heat under the assumption that the optimal trajectory regularity condition [3,4]. In the above problem, the regularity condition is equivalent to

$$\frac{\partial n_\Sigma}{\partial C_y} \ne 0, \; n_\Sigma = N. \qquad (10)$$

In this case, the maximum principle is as follows:

$$\Pi = P_\theta \dot{\theta} + P_H \dot{H} + P_V \dot{V} + P_L \dot{L} + P_Q \dot{Q}, \qquad (11)$$

$$L_1 = \Pi - \lambda(t)\left(n_\Sigma - N\right), \qquad (12)$$

$$P_\theta^\bullet = -\frac{\partial \Pi}{\partial \theta}, \quad P_V^\bullet = -\frac{\partial \Pi}{\partial V}, \quad P_H^\bullet = -\frac{\partial \Pi}{\partial H}$$
$$P_L^\bullet = -\frac{\partial \Pi}{\partial L}, \quad P_Q^\bullet = -\frac{\partial \Pi}{\partial Q}. \quad (13)$$

Here $\lambda(t)$- the Lagrange multiplier, which is determined from the condition of Bliss [3, 4].

$$\frac{\partial \Pi}{\partial C_y} - \lambda(t)\frac{\partial n_\sum}{\partial C_y} = 0 . \quad (14)$$

Π - Pontryagin function, L_1 - Lagrange function.

$P_\theta, P_V, P_H, P_L, P_Q$ - corresponding conjugate variables. For inequality constraints (2) satisfies the complementary slackness.

$$\lambda(t)(n_\sum - N) = 0 . \quad (15)$$

Since the system (1)- (7) is autonomous and the descent no restrictions, the Pontryagin function (11) is identically zero, ie

$$\Pi(P,x,u) \equiv 0, \ u = C_y, \ x = (\theta, V, H_y, L),$$
$$P = (P_\theta, P_V, P_H, P_L, P_q). \quad (16)$$

Conjugate variable $P_Q(t)$ normalized by the condition:

$$P_Q(t) \equiv \text{-1}. \quad (17)$$

The initial conditions for the system (13) and are unknown parameters of the problem. Condition $P_Q(t) \equiv$ -1 and $\Pi(P,x,u) \equiv 0$ is essentially determined by three free parameters:

$$P_\theta(0) = C_1, \ P_V(0) = C_2, \ P_L(0) = C_3 \quad (18)$$

since $P_H(0)$ is determinedfrom the condition $\Pi(P,x,u) \equiv 0$.

In this case,the number of controlled functions at the end of the trajectory (9) coincides with the number of free parameters of the problem (1) - (9), (11), (12), because the time T is not fixed and is a free parameter.

According to the principle of maximum control program chosen from the condition:

$$\Pi \to \max_{C_y} \ khi \ \ Q(T) \to \min \quad (19)$$

We write down the part Pontryagin function (11), which clearly depends onthe control $C_y(t)$

$$\Pi_0 = P_\theta \frac{C_y \rho V S}{2m} - P_V \frac{C_x \rho V^2 S}{2m} . \quad (20)$$

$C_y(t)$ can take control not only limit values (3), but also an intermediate, which is determined from the condition

$$\frac{\partial \Pi_0}{\partial C_y} = 0, \ C_y^* = \frac{P_\theta}{2kP_V V} \ \ C_y^{\min} \le C_y^* \le C_y^{\max} . \quad (21)$$

We calculate three values of the function Π_0 in (20)

$$\Pi_1 = \Pi_0(C_y^{\min}), \quad \Pi_2 = \Pi_0(C_y^{\max}), \quad \Pi_3 = \Pi_0(C_y^*).$$

and

$$\Pi_0^{\max} = \max\{\Pi_1, \Pi_2, \Pi_3\} . \quad (22)$$

Equation (22) determine the nature of the optimal control problem of Pontryagin, ie provided that $n_\sum < N$. Solution of the problem is greatly simplified if the right end of the trajectory is controlled by the condition:

$$H(T) = H_1. \quad (23)$$

In this case,the solution of (1) - (9) is determined by the boundary conditions

$$\theta(T) = \theta_1, \quad V(T) = V_1, \ L(T) = a. \quad (24)$$

And depends on three arbitrary constants C_1, C_2 and C_3.

Thus, the initial problem is reduced to a three parameter problem (1) - (9), (13), (18), (24), and the optimal control is determined at each point t of the maximum principle (24).

III. Restriction onoverload

In the task difficulty of determining the geometry of the optimal trajectory is the identification of points coming off the disabled $n_\sum = N$.

Note that the total overload (2) has two components n_x and n_y. The first is called a longitudinal overload, and the second - normal.

$$n_y = \frac{\rho V^2 S}{2mg_0} C_y, n_x = \frac{\rho V^2 S}{2mg_0} C_x, n_\sum = \sqrt{n_x^2 + n_y^2} .$$

Instead of limiting (2), we introduce a new restriction

$$|n_y| + n_x \le N_1, |n_y| + n_x - N_1 = \varphi(x,u) \le 0 \quad (25)$$

With an appropriate choice N_1 of the inequality (25) is known to besatisfied constraint (2). This fact follows from

$$N_1 \ge [|n_y| + |n_x|] \ge \sqrt{n_x^2 + n_y^2} . \quad (26)$$

equal sign occurs when $C_y = 0$.

We now compute the derivative of $\varphi(x,u)$ (25) follow C_y

$$\frac{\partial \varphi}{\partial C_y} = \frac{\rho V^2 S}{2mg_0}[signC_y + 2kC_y] . \quad (27)$$

In this case, the Lagrange multiplier $\lambda(t)$ for limiting $\varphi(x,u)\le 0$ (25) is determined by the formula

$$\lambda(t)=\frac{2\left(\frac{P_\theta}{2}-kP_V C_y V\right)g_0}{V\left[signC_y+2kC_y\right]}. \qquad (28)$$

IV. Necessary optimality conditions inthe irregular case

Now consider the case when the optimal trajectory contains an interval, when $n_\Sigma = N$,

And in this interval at some point $\frac{\partial n_\Sigma}{\partial C_y}=0$.

The set of points defined by the equations:

$$\frac{\partial n_\Sigma}{\partial C_y}=0, \qquad n_\Sigma = N. \qquad (29)$$

following [3], we call irregular points. For the problem $\frac{\partial n_\Sigma}{\partial C_y}=0$ at $C_y=0$. For the given problem we can use the results of A. I. Dubovistky and A. A. Miliutin [3, 4]. According to [3, 4] in the presence of irregular points conjugate system of equations are

$$\begin{aligned}
\dot{P}_\theta &= -\frac{\partial \Pi}{\partial \theta},\\
\dot{P}_H &= -\frac{\partial \Pi}{\partial H}+\lambda(t)\frac{\partial n_\Sigma}{\partial H}+\frac{d\mu}{dt}\frac{\partial n_\Sigma}{\partial H},\\
\dot{P}_V &= -\frac{\partial \Pi}{\partial V}+\lambda(t)\frac{\partial n_\Sigma}{\partial V}+\frac{d\mu}{dt}\frac{\partial n_\Sigma}{\partial V}, \qquad (30)\\
\dot{P}_L &= 0\\
\dot{P}_Q &= 0
\end{aligned}$$

Here- $\lambda(t)$ Lagrange multiplier-a $\frac{d\mu}{dt}$ generalized function. For these objects are made complementary slackness condition

$$\lambda(t)\left(n_\Sigma - N\right)=0, \qquad C_y\frac{d\mu}{dt}=0. \qquad (31)$$

From (29) it follows that in their regular point (28) and the conjugate variables will experien ceracing on the values of $\mu\frac{\partial n_\Sigma}{\partial H}$ and $\mu\frac{\partial n_\Sigma}{\partial V}$ when $\mu>0$. This is the essential difference between the case of irregular, where the conjugate variables are continuous functions for mixed class constraints [3, 4].

Besides the conditions (29) - (31) on the optimal trajectory should be the conditions of integrability of the Lagrange multipliers and the normalization condition(non-triviality condition of the maximum principle).

V. Example and numerical result

For more details of the problem, we solve the problem of finding the minimum total heating of space shuttle [7], constants and the boundary conditions are:

$C_y^{\min}$ =-0.5; $C_y^{\max}=0.6$;

$\frac{S}{m}=50000\ km^2kg^{-1}$,

$\rho_0=2.3769.10^{-3}$ kgkm $^{-3}$,

$R=6371.2\ km$; $C_{x0}=0.88$,

k = 0.5 ; $g_0=9.8.10^{-3}\ kms^{-2}$,

C = 20; N = 4 ; $\beta=0.145\ km^{-1}$,

$\theta(0)$= -1.25(deg);V(0) =0.35kms $^{-1}$,

H(0)= 100(km);L(0)= 0(km).

The resultreceivedby using Matlab:

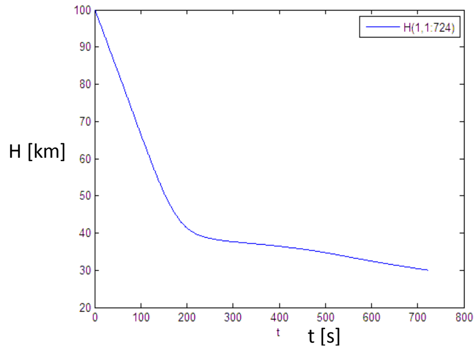

Figure 1.Height H [km]

Figure 1illustrates the shuttle's altitude over time, we see that the height H decreases rapidly from100km down to 40km over a period [0, 200s].

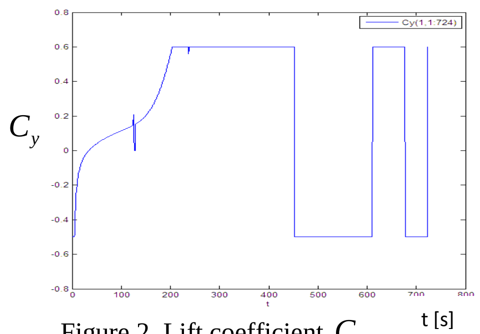

Figure 2. Lift coefficient C_y

In Figure2 illustrates the state of the control variables change over time.

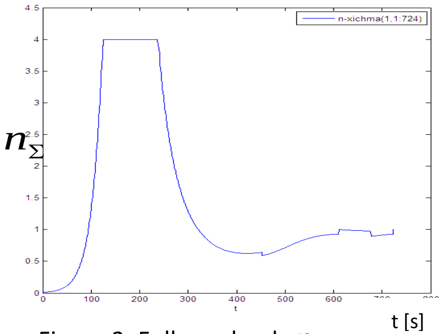

Figure 3. Full overload n_Σ

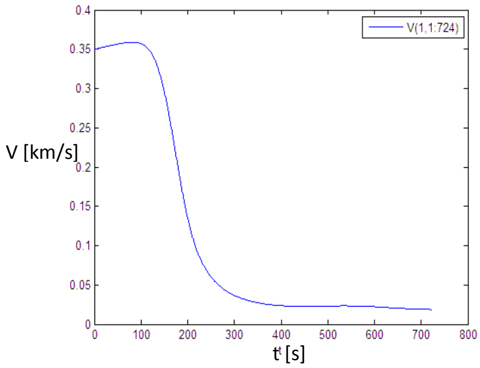

Figure 4.Velocity V[km/s]

Figure 4 shows the velocity of the shuttle also dropped significantly during this period.

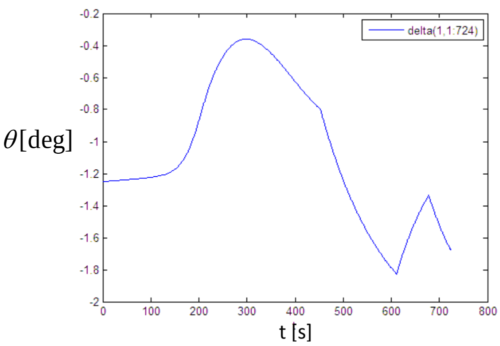

Figure 5. Path angle θ[deg]

Figure 5 describes the orbital inclination angle.

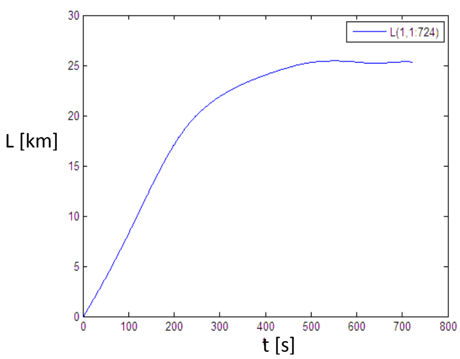

Figure 6. Remote *L* [km]

Figure 6 illustrates the distance of the landing ships over time. We found that the distance L(t) does not increase much after 200 speriod.

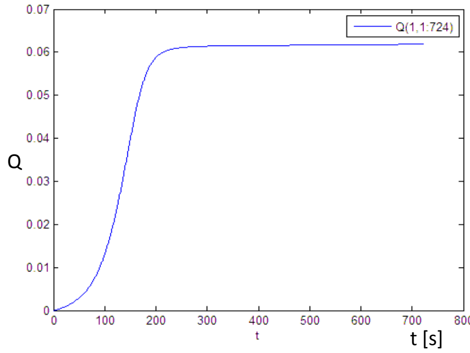

Figure 7. Totalheat Q

Figure 7, in the interval [0, 200s] the total amount of surface temperature increase and stabilize the ship during the period close to landing [200-720s]. According to our simulations on the heat at the surface of the vessel can be considered to have been minimized during landing.

VI. Conclusion

Three parameter boundary value (24), the problem is solvedfor a fixed value *a*. Next, we can choose the desired value a from the minimum of the minimum value of the functional (1).The boundary value problem was solved by the continuation of solutions to the parameter [6].

REFERENCES

[1]. A.E. Bryson and Y.-C. Ho, *Applied Optimal Control*, Rev. Printing (Hemisphere, New York, 1975).
[2]. Ярошевский *В.А.* Вход в атмосферу космических летательных аппаратов. — *М.: Наука,* 1988.
[3]. Афанасьев А.П., Дикусар В.В., Милютин А.А., Чуканов С.В. Необходимое условие в принципе максимума. — М.: Наука, 1990.
[4]. Дикусар В.В., Милютин А.А. Количественные и качественные методы в принципе максимума — *М.: Наука*, 1989.
[5]. O. von Struk and R. Bulirsch. Direct and indirect methods for trajectory optimization. *AnnalsofOperationsResearch* 37(1992)357-373
[6]. Дикусар В. В, Кошька М, Фигура А. Продолжение решений в прикладных задач оптимального управления. *М.,МФТИ*. 2001.
[7]. M. H Breitner and H. Josef Pesch. Reentry trajectory optimization under atmospheric uncertainty as a differential Game. *Advances in dynamic games and applications*, 1994.